# Teachers' Perspectives on the Transition from Secondary to Tertiary Mathematics Education

Ye Yoon Hong
*Suan Sunandha Rajabhat University*
<yyipuni@naver.com>

Suzanne Kerr
*AGGS, Auckland*
<suzannek@slingshot.co.nz>

Sergiy Klymchuk
*AUT University*
<sergiy.klymchuk@aut.ac.nz>

Johanna McHardy
*MAGS, Auckland*
<jmchardy@mags.school.nz>

Priscilla Murphy
*MIT, Auckland*
<priscilla.murphy@manukau.ac.nz>

Sue Spencer
*EGGS, Auckland*
<suespencer1@gmail.com>

Mike Thomas
*The University of Auckland*
<m.thomas@math.auckland.ac.nz>

Peter Watson
*AUT University*
<peter.watson@aut.ac.nz>

The transition from school to tertiary study of mathematics is coming under increasing scrutiny in research. This paper reports on some findings from a project analysing the transition from secondary to tertiary education in mathematics. One key variable in this transition is the teacher/lecturer. This paper deals with a small part of the data from the project— analysing secondary teachers' responses to questions on the differences teachers perceive between school and university and the importance of calculus, a bridging content. The results show that teachers lack a clear understanding of the issues involved in the transition and there is a need for improved communication between the two sectors.

## Introduction

Growing interest in the transition period from school to university has been fuelled by concerns about decreasing numbers of students opting to study mathematics at university and beyond (e.g., the ICMI Pipeline Project), and their apparently decreasing levels of competence (Smith, 2004). Serious concern was expressed in the report *Tackling the Mathematics Problem* commissioned by the London Mathematical Society (LMS, 1995), surrounding a lack of essential technical facility, a marked decline in analytical powers, and a changed perception of what mathematics is especially with regard to the place of precision and proof. This is not restricted to 'new undergraduates' who ten years ago would not have proceeded to higher education. The problem is more widespread with concern over the possibility of a widening gap shown by studies in different countries around the world (e.g., Brandell et al., 2008; Engelbrecht & Harding, 2008). In addition, it is not just the case that some students are less well-prepared, but many 'high-attaining' students may be lacking in fundamental notions of the subject.

The problem of transition from secondary to tertiary education has been recognised for some time, with Guzman et al. writing "…the secondary-tertiary transition can be seen as a major stumbling block in the teaching of mathematics" (Guzman et al., 1998, p. 748). It seems that although mathematics in elementary and high school enjoys a special position in the curriculum, the knowledge and skills of incoming university students may not echo this fact (Artigue, 2001). One possible reason for this is that a number of changes occur in the transition to tertiary education, including those in teaching and learning styles, type of mathematics taught, conceptual understanding, procedural knowledge required to advance through the material, and changes in the amount of advanced mathematical thinking needed.

The amount of research in mathematics education at the tertiary level is still modest (Selden & Selden, 2001), and does not adequately cover the secondary-tertiary transition. However, a review of the literature that exists produces ample

In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides: Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia* (Vol. 1). Palmerston North, NZ: MERGA.

evidence that the transition in mathematics is a complex problem (Barnard 2003; De Guzman et al., 1998; McInnes et al., 2000; Schoenfeld, 1994; Wood 2001). Further, researchers writing on the transition period from school to university education in mathematics also indicate that the mathematical under-preparedness of students entering university is an issue (Hourigan & O'Donoghue, 2007; Kajander & Lovric, 2005; Luk, 2005; Selden, 2005), and the impact this has on students' success in university mathematics (Anthony, 2000; D'Souza & Wood, 2003; Leamson, 1999). One specific problem relates to students' procedural understanding of algebraic material (Kajander & Lovric, 2004; Novotna & Hoch, 2008). They provide a number of reasons for that under-preparedness such as a recent trend of moving from elite to mass university education, lowering the mathematics standards at school and university, inadequate funding, etc.

While not wishing to prejudice the outcomes of the research, our framework is built around the hypothesis that there may be qualitatively different approaches to thinking about mathematics at school and tertiary levels. A developing theory by Tall (2004, 2008) suggests that mathematical thinking exists in three *worlds*, the embodied, symbolic and formal. The embodied is where we make use of physical attributes of concepts, combined with our sensual experiences to build mental conceptions. The symbolic world is where the symbolic representations of concepts are acted upon, or manipulated, where it is possible to switch from processes to *do* mathematics, to concepts to *think* about mathematics. The formal world is where properties of objects are formalised as axioms, and learning comprises the building and proving of theorems by logical deduction from these axioms. If tertiary courses are trying to build thinking in the formal world with students who are primarily symbolic thinkers, then difficulties will arise. Researchers such as Ball (2002) and Ma (1999) have argued that a deeper understanding of *why* mathematical ideas work rather than just *how* they are carried out is crucial to retention and long-term understanding. In addition, many students are exposed to a formal deductive approach in mathematics for the first time on entry to university and may therefore experience a significant amount of cognitive conflict in their first year (Tall, 1997).

## Method

This study is part of a much larger research project entitled 'Analysing the Transition from Secondary to Tertiary Education in Mathematics' involving teachers, lecturers and students that employs questionnaires, interviews and teaching observations. A questionnaire was sent to all 350 secondary schools in New Zealand to be completed by all teachers who teach calculus in Years 12 or 13 (age 17-18 years). The questionnaire was posted, complete with a stamped addressed return envelope and teachers were given three weeks to answer. After this a follow-up copy was sent by email to remind teachers to reply. Using this approach we received a total of 178 responses, and some of these were interviewed. There are no figures available on the total number of calculus teachers in the schools, which vary in size from fewer than 30 students (small country school) to 3000 (inner city), but we estimate the response rate at about 30% of the population. In this paper we present and analyse teachers' responses to two questions from the questionnaire and some Likert-style questions, along with some interview comments. Of the 178 teachers who responded to the survey, only 154 respondents gave personal demographic details. Of these 82 (52%) were male and 79 (48%) female. The majority was in the 41-50 (35%) or 51-60 (29%) age groups and 90% had English as a first language. Also 55% of them had taught calculus for more than 11 years.

## Results

In Q22 of the survey teachers were asked if they thought that there were differences between Year 13 and first year tertiary calculus teaching in the following areas: assessment, teaching style, teaching resources, teaching emphasis, technology use, teacher preparedness, and students' experiences, and if so why. An overview of the responses is summarised in Figure 1. While there are perceived differences by more than 30% in assessment, teaching style, teaching resources and student experiences, the most common response was that teachers answered, "don't know" whether there are any differences. This could be of concern when considering the transition from school to tertiary study since it implies a lack of knowledge of the tertiary situation. Three teachers alluded to the possible reason for this in their interviews.

> I think that we don't.. we haven't got a lot of uniformity amongst schools in presenting to students what to expect at university, and I don't think the universities do that brilliant a job in feeding back to schools what they want…I do believe that, where schools are trying to find out what's required at university. (T018)

> I think it would be really useful from my point of view to actually meet with a lot of the people who get our students, and say to them 'what are the things that we are doing that are really good and what are the things that you would like us to do more of?' (T010)

> I wouldn't know. The task of a secondary school is to follow the curriculum. Occasionally I would divert and teach something slightly different, beyond the curriculum, but I don't do it often. (T146)

*Figure 1.* Percentages responses for each part of Q22.

### *Assessment*

Those who went on to comment about differences in assessment between school and tertiary level made observations such as, "A lot more assessment" (4, 6.1%), although it is not clear whether they felt that school or tertiary had more. References were made to the differences in assessment styles, such as "Standards-based versus norm-referenced" (4, 6.1%) and "Presumably universities are not using the type of marking used in NCEA [national] exams." (2, 3.0%). There were a few comments about the use of technology in assessment; "Emphasis on no calculator use in tertiary calculus." (2, 3.0%) and "University has computer lab sessions and assignments done using computers." There were also interesting differences in the perception of relative difficulty, “Easier to achieve at a top level at university than at NCEA level 3”; “University level is a bit more challenging”; and “Some topics requiring higher skills and extra topics such as matrix and learning how to solve using Matlab.” In their interview, teachers talked at length about the NCEA assessment and the attitudes of students and themselves in dealing with this summative assessment. A theme of tailoring work to assessment at a specific, often lower, level was prevalent.

> The type of questions I give them is similar to that they will receive in an exam or an assessment. (T145)

> I think that the internal assessments...because you know what you’re going to be assessing them and because of time constraints, you can teach the content that’s in the assessment. I’m afraid that that’s the sort of thing that has crept in. (T156)

> Let me think of an example, let us go back to my expectations with the majority of the class, if I’m aiming at achieved or merit I might skip out the excellence part work at the end. (T134)

As a consequence, it is not surprising to see this attitude reflected by students, to the concern of some teachers.

> It’s a different system, NCEA they can say oh that’s an excellence question, I won’t worry about that, I’ll just work at merit/achieved. Whereas at university they have to do the whole lot, and so in that way NCEA can be a problem for students that want to work a lower level and are thrown hard problems at university. (T112)

> I have seen the students going more for achieved level, and no one, not many of them are working at excellence level. (T122)

### *Teaching Style*

The prevalent perception of differences in teaching styles was that the level of interaction between lecturers and students at tertiary level is not sufficient (41, 64.1%). Comments such as “Tertiary students are taking more responsibility for their own learning. Teaching style is more teacher-centred” and “less personal interaction with students” were common, but this was also tempered with, “Attendance at tutorials at university may alleviate or moderate the ‘clinical’ nature of lecturing.”, “Large course sizes at university prevent interactive investigative approach to new material. Difficult for students to ask questions” and “more lecturing rather than teaching.” Other observations included, “Teaching is more detailed in Year 13 than in the first year tertiary calculus” and “more technology and lecturing” at tertiary level. Teachers indicated that the communication between themselves and the students was of paramount importance to them, and there were many comments in the interviews that supported that.

### *Teaching Resources*

The perception of the teachers (20, 64.5%) was that tertiary institutions have far more resources than secondary schools. Examples given included: CD resources, textbooks and access to computers. “Universities usually have much better computer resources” enabling “Mostly technology based” courses (2, 6.5%) and, “A lot of good clearly explained textbooks.” However, the interviews predominantly highlighted that the availability of time was the greatest resource lacking at schools. Second to this was the lack of availability of resources such as computers and software.

### *Teaching Emphasis*

It is significant that 71.1% answered that they didn’t know of any differences in teaching emphasis. Those who commented mostly felt there was greater depth to the understanding (2, 11.1%), an emphasis on the theory, and a more formal approach (2, 11.1%) at tertiary level than at Year 13. Some felt there were "Different approaches to certain sections, inclusion/exclusion of topics at school" (2, 11.1%), and "more on pure mathematics (and) less on applications." One teacher spoke at length in a negative way about the change in emphasis to practical contexts with the introduction of NCEA assessment.

> …students who are less capable struggle to understand maths unless they can fit it into a practical situation then everything we do needs to have a direct link to a practical situation, and that isn't mathematics in my view…If we keep on going this way… then we'll actually lose what mathematics offers, because it will become so simplistic … And that's scary for me, that it's all going that way, that we're getting out of the theoretical mathematics. (T159)

## *Technology*

Those who commented about the use of technology e.g. computers, PowerPoint, Matlab, and projectors (17, 50%) found the tertiary use of technology much greater than that of Year 13 teachers except for the use of graphic calculators: "GC (graphics calculator) is not used at university" (6, 17.6). "Tertiary - also online access of notes, assignments, use computer programmes." (2, 5.9%) "Vast resources" (2, 5.9%). This could possibly be due to the curriculum requirements in each case. The main discussion areas in the interviews were about the lack of access to computers, the increasing use of data projectors (PowerPoint) in school teaching and the use of graphic calculators. A concern about a lack of understanding when using the graphic calculator was talked about.

> My belief is that in calculus or senior mathematics we are trying to help them become analytical thinkers… I think that calculators are undermining what I'm trying to get through in a subject like calculus. (T159)

> I haven't personally used a lot of technology apart from calculators, partly because of difficulty in getting time on the computers. Programmable calculators allow the students to do problems that they don't really understand. (T145)

> The technology I use is: data projector, I use the overhead projector, we've got the graphics calculators, computers for generating simulations and – yeah the [inter]active whiteboard is in my room but so far I haven't learnt how to use it. (T010)

> It [technology] has a significant role in teaching and learning and I have some reservations about the use of technology in assessments, because I am concerned that technology can mask real knowledge. (T146)

## *Teacher Preparedness and Support for Teachers*

Again the predominant reply in this section was that teachers 'don't know' (78.9%, $N$=141). However, in the interviews, the teachers discussed this topic at length with the predominant issue being time and workload—particularly administrative workload. "If you're tired and you're wrapped off your feet because you're doing your reports and ninety thousand other things.., you don't prepare." (T156); "My workload definitely affects the way I teach." (T018) and "The workload affects my teaching to the extent that I'm not entirely happy with the quality of teaching I've been able to do" (T010). One of the key determinants of having the energy to prepare well was the issue of classroom management and student control. "Preparation time; behaviour of the students and the lack of respect… A lot of your class time is spent on managing class." (T145); and "…the teaching time is just about 5 or 10 minutes during a period, and the rest is spent on giving them some tasks that they have to do on their own just to keep them quiet." (T122). The perception was that tertiary lecturers had more time to prepare and also had more time to work together at departmental level. "Possibly more support/preparedness at university and perhaps time." (2, 13.3%) "University has more access to support for resource preparation." "More colleagues and departmental discussion at university. Less pedagogy-driven and more mathematics-driven at university."

### *Students' Experiences*

There was a perception that there is more teacher-student interaction at school (25, 58.1%) than at tertiary level, supported by the following comments: "Closer teacher/student relationship at school." "Suspect that teacher-student communication would be considerably less at tertiary level." "More self-motivated in university" and "Tertiary students studying maths are usually more motivated than Y13 students." (10, 23.3%). The view was expressed that lecturers don't need to take responsibility for their students' results but secondary school teachers see their role as supporting students through the learning process and giving them opportunities to revisit work.

### *Transition*

Following a series of questions to ascertain teachers' perceptions about the differences between Year 13 and first-year tertiary calculus, teachers were specifically asked if they thought students had problems moving from school to tertiary calculus, and what could be done to make the transition smoother. 154 teachers answered that question with about half saying they didn't know and the Yes and No answers about a quarter each. Sixty-nine teacher respondents had opinions about making the transition easier. The most often repeated suggestion (10% of the respondents who gave further ideas) was that "Students should aim higher to get merit or excellence as the tertiary education assumes they have a sufficient knowledge of Yr 13 calculus." It appears that these teachers observe students simply aiming to 'pass' rather than understand at a deeper level. Related to this is the observation "Study skills and self-discipline is the main requirement".

Nearly 9% believed that the amount and quality of interaction between lecturers and students was a problem, mentioning 'one-to-one contact and help'. Linked to this is the perceived difference between teaching styles at school and university. Respondents think that one of the difficulties for students is the sheer quantity of information given in a lecture situation that leads to a lack of time to understand properly. "I think students are used to understanding before moving onto the next topic. In a lecture, the quantity is greater so they just copy instead of trying to understand what's being taught." One suggestion was that lecturers change their approach: "Train lecturers to organize material and have better understanding of student knowledge." More that 7% believe that more communication between school and university would ease the transition for students. However, one teacher was unconcerned about this: "The transition is a change of learning cultures from hands-on to hands-off—all part of the learning curve."

More than one respondent mentioned that if calculus is well taught at school, the first year of university calculus can be 'too easy'. "Only if it were properly taught at school first year university mathematics is sometimes easier than L3 maths and there is little challenge for the top students in first year. As a consequence, second year exams are a bit of a shock."

*Preparedness and Importance of Calculus*

In addition to Q22, three items on a five-point Likert scale (1 = strongly disagree, 5 = strongly agree) asked teachers to rate how well prepared their students are for further study in mathematics (positive response corresponding to students being well prepared). The mean score here was 3.8 (see Table 1), indicating that the teachers mostly felt that their students are well prepared for further study. Four items on the same Likert scale asked teachers to rate the importance of calculus (a positive response corresponding to calculus being important). Here the mean score was 3.9, showing that calculus is generally regarded by teachers as being important in society. Specific questions relating to teachers opinions produced interesting results. Close to 90% of the teachers believe that calculus is of importance in society, and although 86% of the 170 teachers who responded encourage their students to study mathematics further after school and only 6% (of 167) believe the students are not well prepared for this, teachers generally do not know how calculus is taught at tertiary level, with 60% not knowing whether calculus is taught differently at tertiary level compared with schools and 31% believing it is taught differently. There is apparently a need to educate teachers about the requirements of tertiary calculus.

Table 1
*A Summary of the Likert Scale Responses*

| Question | Mean | SA | A | N | D | SD |
|---|---|---|---|---|---|---|
| I encourage my students to study mathematics further after school. (*N*=170) | 4.17 | 63 (37.1%) | 83 (48.8%) | 17 (10%) | 4 (2.4%) | 3 (1.8%) |
| My students are well prepared for studying further calculus after school. (*N*=167) | 3.91 | 33 (20%) | 77 (46.7%) | 45 (27.3%) | 8 (4.9%) | 2 (1.2%) |
| Calculus is taught differently at tertiary level from school. (*N*=169) | 3.27 | 9 (5.8%) | 39 (25.3%) | 92 (59.7%) | 12 (7.8%) | 2 (1.3%) |
| Calculus is of little benefit in society. (*N*=169) | 1.68 | 6 (3.6%) | 3 (1.8%) | 13 (7.7%) | 56 (33.1%) | 91 (53.8%) |
| Calculus is of major importance in the real world. (*N*=166) | 3.83 | 45 (26.6%) | 75 (44.4%) | 32 (18.9%) | 10 (5.9%) | 7 (4.1%) |
| I consider calculus as a core course in my students' programme. (*N*=169) | 3.69 | 24 (14.7%) | 79 (48.5%) | 48 (29.4%) | 10 (6.1%) | 2 (1.2%) |
| NCEA Level 3 has too much emphasis on calculus. (*N*=171) | 2.13 | 6 (3.8%) | 8 (5%) | 31 (19.4%) | 71 (44.4%) | 44 (27.5%) |

In summary, the research shows that the main differences as perceived by teachers between Year 13 and first year tertiary calculus teaching were that there is a greater use of technology at tertiary; tertiary institutions are better resourced particularly in terms of computers and time; there is a more formal approach to the teaching at tertiary; secondary teachers interact more with their students; secondary teachers spend a large amount of time on administration at the expense of lesson preparation and there is more teaching to the assessment at school due to NCEA. Overall there was a great deal of ignorance expressed about tertiary calculus teaching. Clearly there are important roles for secondary teachers and tertiary lecturers to play in helping students with their transition. One aspect of this is the need for closer communication between secondary and tertiary teachers.

# References

Anthony, G. (2000). Factors influencing first year students' success in mathematics. *International Journal of Mathematical Education in Science and Technology*, *31*(1), 3-14.

Artigue, M. (2001). What can we learn from research at the university level? In D. Holton (Ed.), *The teaching and learning of mathematics at the university level: An ICMI study* (pp. 237-254). The Netherlands: Kluwer Academic Publishers.

Ball, D. (2002) Towards a practice-based theory of mathematical knowledge for teaching. In B. Davis and E. Simmt (Eds.), *Proceedings of the Canadian Mathematics Education Study Group Conference.* (pp. 3-14). Kingston, Ontario.

Barnard, D. (2003). The transition to mathematics at university: Students' views. *New Zealand Journal of Mathematics*, *32* (Supplementary Issue), 1-8.

Brandell, G., Hemmi, K., & Thunberg, H. (2008). The widening gap – a Swedish perspective. *Mathematics Education Research Journal*, *20*(2), 38-56.

D'Souza, S., & Wood, L. (2003). Rationale for collaborative learning in first year engineering mathematics. *New Zealand Journal of Mathematics*, *32* (Supplementary Issue), 47-55.

Engelbrecht, J., & Harding, A. (2008). The impact of the transition to outcomes-based teaching on university preparedness in mathematics in South Africa, *Mathematics Education Research Journal*, *20*(2), 57-70.

Guzman, M., Hodgson, B., Robert, A., & Villani, V. (1998). Difficulties in the passage from secondary to tertiary education. *Proceedings of the International Congress of Mathematicians, 3*, 747-762.

Hourigan, M., & O'Donoghue, J. (2007). Mathematical under-preparedness: The influence of the pre-tertiary mathematics experience on students' ability to make a successful transition to tertiary mathematics courses in Ireland. *International Journal of Mathematical Education in Science and Technology*, *38*(4), 461-476.

Kajander, A., & Lovric, M. (2004). The double cohort: preliminary research on the transition from secondary to university level mathematics. *The Ontario Mathematics Gazette*, *42*(4), 33-36.

Kajander, A., & Lovric, M. (2005). Transition from secondary to tertiary mathematics, McMaster University experience. *International Journal of Mathematical Education in Science and Technology*, *36*(2-3), 149-160.

London Mathematical Society (LMS, 1995). *Tackling the mathematics Problem*. Retrieved August 6, 2007, from http://www.lms.ac.uk/policy/tackling_maths_prob.pdf.

Luk, H. S. (2005). The gap between secondary school and university mathematics. *International Journal of Mathematical Education in Science and Technology*, *36*(2-3), 161-174.

Ma, L. (1999) *Knowing and teaching elementary mathematics*. Mahwah, New Jersey: Lawrence Erlbaum.

McInnes, C., James, R., & Hartley, R. (2000). *Trends in the first year experience in Australian universities.* Canberra. Australia: AGDS.

Novotna, J. & Hoch, M. (2008). How structure sense for algebraic expressions or equations is related to structure sense for abstract algebra, *Mathematics Education Research Journal*, *20*(2), 93-104.

Selden, A., & Selden, J. (2001). Tertiary mathematics education research and its future. In D. Holton (Ed.), *The Teaching and Learning of Mathematics at the University Level: An ICMI Study* (pp. 207-220). The Netherlands: Kluwer Academic Publishers.

Selden, A. (2005). New developments and trends in tertiary mathematics education: Or more of the same? *International Journal of Mathematical Education in Science and Technology*, *36*(2-3), 131-147.

Schoenfeld, A. H. (1994). What do we know about mathematics curricula? *Journal of Mathematical Behaviour*, *13*(1), 55-80.

Smith, A. (2004). *Making mathematics count*. UK: The Stationery Office Limited.

Tall, D. O. (1997). From school to university: The transition from elementary to advanced mathematical thinking. In M. O. J. Thomas (Ed.), *Proceedings of the 7th Conference of the Australasian Bridging Mathematics Network* (pp. 1-20), Auckland, New Zealand.

Tall, D. O. (2004). Building theories: The three worlds of mathematics. *For the Learning of Mathematics, 24*(1), 29-32.

Tall, D. O. (2008). The transition to formal thinking in mathematics. *Mathematics Education Research Journal*, *20*(2), 5–24.

Wood, L. (2001). The Secondary-Tertiary Interface. In D. Holton (Ed.), *The Teaching and Learning of Mathematics at University Level: An ICMI Study* (pp. 87-98). Holland: Kluwer.